A Coal Mining Production Function

Appendix C

to

A Special Report to the Senate Finance Committee

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Introduction

This model provides an integral production simulation tool for the model of Coal Supply and Demand offered in Appendix B. This model evaluates the *economies of scale* within underground coal mining and the *economies of scope* across surface and underground mining in nine southern West Virginia counties. The results of this model provide simulation of production changes in underground mining resulting from regulatory impact on surface mining.

This effort permits an overall output simulations of a variety of regulatory impacts that potentially impact surface coal production. This was primarily the *economies of scope* contribution to the study which measured the impact of a decline in surface production on underground production. This is theoretically justified from a variety of models which identify the existence of non-separable cost functions. The formalization of this theory is attributed to Baumol, Panzar, and Willig [1983]. A simplification of their approach involves the production of two goods, x and y; the production costs of which may be described by the function $C(x,y)$. The existence of the economies of scope is confirmed by the relationship:

$$C(x,y) < C(x,0) + C(0,y)$$

where the cost of producing the two goods together is less than a separation of the production process. Testing this hypothesis and generating simulation results are a primary goal of this research.

The Model

In order to test this relationship we model not the cost function, but the production function. This function relates the combination of inputs to the combination of outputs in a regional setting. This is especially useful for our purposes since we are focusing on county level, not firm level outputs. The absence of firm or regional specific production costs and capital costs also recommends the use of the production function approach. The use of a production function in lieu of a cost function follows from an extensive duality result.[1] Assume a cost function that is differentiable, concave, monotonic, and homogeneous of degree one. Establishing two inputs,

[1] For an expanded discussion see Varian, [1992, pg. 82-93. And Silberburg [1990] pg. 281-284.

capital (K) and labor (L), respective factor prices, w_1 and w_2, output x and technological adjustment parameter a we have:

(Equation Set C-1)

$C(w,x) = xw_1^a w_2^{1-a}$ then individual input demands (capital and labor) are derived from:

$$K(w_1,x) = axw_1^{a-1}w_2^{1-a} = ax\left(\frac{w_2}{w_1}\right)^{1-a}$$

$$L(w_2\ ,x) = (1\text{-}a)xw_1^a w_2^{-a} = (1\text{-}a)x\left(\frac{w_2}{w_1}\right)^{-a}$$ which results in:

$$\left(\frac{K}{ax}\right)^{\frac{1}{1-a}} = \frac{w_2}{w_1} = \left(\frac{L}{(1-a)x}\right)^{\frac{-1}{a}}$$ and:

$$\frac{K^{-a}}{a^{-a}x^{-a}} = \frac{L^{1-a}}{(10a)x^{1-a}}$$ which can be rewritten

$$\left[a^a(1-a)^{1-a}\right]x = K^a L^{1-a}$$ which is the well known Cobb - Douglas Production Function

The production function method is straightforward and test (among other items) simply if underground production is affected by the presence of surface mining in the same county. The model in general form takes the form:

$$Q_u = f(K_i, L_i, Q_s)$$

where Q_u is underground production, K is productive capital and machinery, L_i is county level employment in underground mining and Q_s is county level output in surface mining. The specification of this model is theoretically straightforward. However a number of data restrictions complicate the process.

The Data

The short time period of available data recommends a cross sectional time series estimation technique to preserve degrees of freedom. This was complicated by the absence of county specific capital proxies. The product of the total capital and capital utilization rates in the underground bituminous coal mining industry was generated to serve as an aggregate proxy for capital. This measure was employed by Naples [1998] for a coal industry production function. Underground and surface quantities and underground mining employment data were available at the county level. The prime modeling drawback to this technique is that it limits that interpretation of the technology parameters in the Cobb-Douglas Production function. The usual interpretation of the technology parameters (the *a* component) is that the sum of these component reflect the economies of scale. Since we will perform both disaggregated and aggregated analysis this interpretation is problematic. Control variables listed below were also employed in the specification of this production function. See Table C-1.

Table C-1
Production Function Variables

Variable	Description
Qu, Qs	county level coal production in tons (underground and surface)
Uemp	county level underground mining employment
Capuse	The product of the national capacity utilization rate and available capacity in the underground mining sector

Data was collected from the *Energy Information Administration*, *West Virginia Office of Miner Health, Safety and Training* and the *West Virginia Coal Association*. These data are publicly available, and most were confirmed by multiple sources.

Econometric Methods

As in the model presented in Appendix A it became apparent that *ordinary least squares* estimates would be inappropriate for a variety of reasons. A substitute for *ordinary least squares* is a *weighted least squares* estimator that minimizes a weighted horizontal and vertical deviation from the estimated linear function. The *weighted least squares* estimator appears as:

(Equation Set C-2)

$$B_{wls} = \left(X'V^{-1}X\right)^{-1} X'V^{-1}y \quad \forall V^{-1} = [\ddots]\left(1/s_{11},\ldots,1/s_{n}\right.$$
$$where\ \mathrm{var}\left(B_{wls}\right) = \left(X'V^{-1}X\right)^{-1}$$

The *weighted least squares* estimator is efficient and consistent, but not asymptotically unbiased in a single equation model with autocorrelated or heteroscedastic errors (see Kmenta, 1986; Kennedy, 1996). This presents additional problems which we discuss later.

The use of a panel series with a number of cross sectional invariant parameters was immediately considered and subsequently adopted. The panel technique selected was the fixed effects model.[2] Similarly, following a visual inspection of the data a first differenced, or de-trended estimation technique appeared appropriate. This was confirmed through an exhaustive set of unit-root tests.[3] Similarly, a log-log specification was initially employed for its ease of interpretation (see Varian, 1992; Greene, 1994; Kennedy, 1996).

Deviations from the classical linear model also included the potential for autocorrelated errors, multi-collinearity and heteroscedastic errors. The first two problems were not apparent Thus eliminating inconsistent errors in the weighted least squares estimator. A similarly easy step was the use of White's heteroscedasticity invariant standard errors in estimation:

[2]The Fixed effect model is appropriate when exhausting the study population, as we have done. Other reasons including autoregressive componeents recommend this choice, with no reasonable substitutes emerging.

[3]The augmented Dickey-Fuller tests clearly rejected the hypothesis of a unit root meaning that these variables possessed a time trend, or were non-stationary. The hypothesis of a unit root in first differences for each variable could not be rejected at high levels of significance, typically .01 percent. The authors will provide these results upon request.

(Equation C-4)

$$X_W = \frac{T}{T-K}\left(X'X\right)^{-1}\left[\sum_{t=1}^{T} u \right]$$

This matrix, $\mathbf{X_W}$, is employed to calculate the standard errors. This removes the inefficiencies noted in the *weighted least squares* estimator under conditions of heteroscedastically distributed errors, see White (1980). This cleared the final hurdle. All of these empirical procedures were programmed as an *a priori* step in estimation.

Estimation Results

The form of the model is:

(Equation C-5)

$$\log Q_u^i - \log Q_{u,t-1}^i = \alpha_i + \beta_1 [\log(CAPUSE) - \log(CAPUSE_{t-1})] +$$

$$\beta_2^i [\log(Q_s^i) - \log(Q_{s_{t-1}}^i)] + \beta_3^i [\log(Uemp) - \log(Uemp_{t-1})] + e_{i,t}$$

We placed no parameter restrictions on this model. Suppressing the logarithmic and first differenced forms the results appear in Table C-2 and C-3.

Table C-2
Parameter Estimation Results

Variable	Coefficient	Standard Error	T-Statistic	Probability
Capacity Utilization	0.816308***	0.213611	3.821475	0.0002
Boone, Surface Q	0.705788***	0.161316	4.375185	0
Fayette, Surface Q	0.160822	0.466682	0.344608	0.731
Kanawha, Surface Q	0.209531	0.227753	0.919993	0.3594
Logan , Surface Q	0.29278Ü	0.182453	1.604684	0.1111
McDowell, Surface Q	0.003671	0.010007	0.366866	0.7143
Mingo, Surface Q	0.354241Ü	0.292016	1.213086	0.2274
Raleigh , Surface Q	0.071852	0.043741	1.642658	0.103
Wyoming, Surface Q	0.071419	0.053293	1.340131	0.1827
Boone, Underground Employment	0.525092***	0.131444	3.99478	0.0001
Fayette, Underground Employment	0.724972***	0.109545	6.618055	0
Kanawha, Underground Employment	0.56212***	0.112726	4.986583	0
Logan, Underground Employment	0.446617***	0.146244	3.053914	0.0028
McDowell, Underground Employment	0.618985***	0.060017	10.31349	0
Mingo, Underground Employment	0.567561*	0.335099	1.693709	0.0928
Raleigh, Underground Employment	-0.025063	0.057273	-0.437604	0.6624
Wyoming, Underground Employment	0.346107**	0.201437	1.718191	0.0883

*** - Significant at the 1% level.
** - Significant between the 1% and 5% levels.
* - Significant at the 10% level.
Ü - Significant between the 10% and 15% levels.

Table C-3
Intercept and Test Statistics

Fixed Effects Intercept			
Boone	0.012162		
Fayette	0.054317		
Kanawha	0.013269		
Logan	-0.038316		
McDowell	0.031652		
Mingo	0.005035		
Raleigh	0.009308		
Wyoming	-0.015266		
R-squared	0.56	**Mean Dependent Variable**	-0.027929
Adjusted R-squared	0.47	**Standard Deviation of Dependent variable**	0.24499
Standard Error of regression	0.178136	**Sum squared resid**	3.934812
Log likelihood	75.90205	**F-statistic**	9.74586
Durbin-Watson statistic	1.717796	**Prob(F-statistic)**	0

The quality of the model test statistics is heartening. For our purposes, interpretation of the capital coefficient is not critical. The technical parameter, which is the coefficient estimate for the capital and county level employment variables for underground mining is not robustly interpreted because of the differences in aggregation. However, it does appear that *economies of scale* in underground mining is censored. That is the sum of the technology parameters B_1 and B_3 are in every statistically significant instance greater than unity.

(Equation C-6)

$$iff \quad \beta_1 + \beta_2 = \begin{cases} > 1 & \text{Increasing Returns to Scale} \\ = 1 & \text{Constant Returns to Scal} \\ < 1 & \text{Decreasing Returns to Scale} \end{cases}$$

The existence of increasing returns to scale is synonymous with economies of scale. This condition suggests that the scale of production is limited by the geological distribution of coal, not the physical mix of inputs. This is important for it suggests that cost reducing changes in firm size are not possible. Average costs of production are dictated by the size of the coal seam, not the mix of workers and capital. This interpretation is supported by the data on mine size and price. The county level intercepts are not important for our purposes, as their most common interpretation is to scale the output of these counties. The inclusion of insignificant variables was retained for illustrative purposes. The model is not sensitive to zero restrictions on these parameters.

The coefficient estimates on the surface mining output evaluates the *economies of scope* in production across surface and underground mining in this model. The critical observation across each is the positive sign on each. Though the standard treatment of statistical significance is absent for 7 of the 9 counties there is clearly a pattern of economies across the two types of mining operations. This is the purpose of this modeling effort. Note that economies of scope are treated very similarly in this instance to economies of scale. Here however, the parameter estimate B_2 is interpreted as the economies of scope determinant.

(Equation C-7)

$$iff \quad \beta_2 = \begin{cases} > 0 & \text{Economies of Scope} \\ = 0 & \text{Neutral or Unrelated Economies of} \\ < 1 & \text{Diseconomies of Scope} \end{cases}$$

Conclusions

The standard duality result from a cost function yields a Cobb-Douglas form production function which we have estimated. The economies of scale interpretation suggest important conclusions regarding the overall ability of firms to modify size in response to price changes. They cannot adjust output levels at a particular seam without increasing per unit costs. The economy of scope conditions suggests that reduction of one type of production (here we have treated surface mining as the likely target of regulatory reduction) potentially affects the productivity of underground mining.

Both observations suggest that the cost reducing options for firms must be based on worker or capital productivity from exogenous sources, not through modification of scale. Second, the existence of economies of scope suggests that, perhaps counter-intuitively, that the loss of surface mining through regulatory intervention could well cause a reduction in underground mining as well.