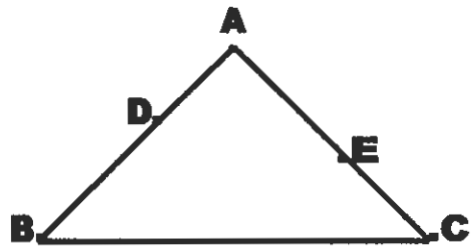


Marshall University Mathematics Competition
2004

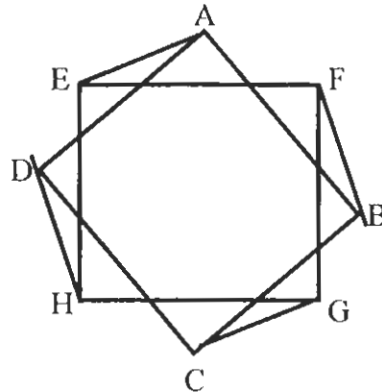
1. A pancake restaurant serves six circular pancakes of different sizes in each serving. Unfortunately, the chef is somewhat careless and just puts the six pancakes on the plate in any order. As the server, you need to turn the first two or three or four or five or six pancakes in the stack with a spatula to get the pancakes in increasing order by size. Suppose the chef hands you a plate with the pancakes stacked 241635 where 1 represents the smallest and 6 the largest pancake. You could, for example, flip over the first two to get 421635, the first three to get 142635, the first four to get 614235, the first five to get 361425, or all six to get 536142.
- a) Starting with 241635 describe a sequence of flips that will result in a properly stacked plate of pancakes (123456).
- b) Outline a method that will straighten out any stack of pancakes in 10 or fewer flips.

2. $\triangle ABC$ is isosceles with $\overline{AB} \cong \overline{AC}$, $m(\angle BAC) = 20^\circ$, $m(\angle BCD) = 60^\circ$, and $m(\angle CBE) = 50^\circ$. What is $m(\angle BED)$?



3. This instruction accompanied a smoke detector: "Mount the unit on the ceiling, at least one foot from any wall, and three feet from any corner." (Smoke could take extra time to reach detectors mounted near walls and/or corners, giving less time for escape.)
- Suppose you plan to mount a smoke detector in a rectangular room that is twenty feet long and fifteen feet wide. You are very afraid of fire so you decide to be more conservative than the instruction that came with the detector. You are going to mount the detector so that it is three feet from any wall and five feet from any corner. What is the area of the permissible region within which you could mount the detector?
- a) Give a rough estimate of the permissible area, to the nearest whole square foot. Be sure to justify your estimate. You **need not** justify that it is within one square foot.
- b) Give the exact value of the permissible area. Be sure to justify this value clearly.

4. An airline serves eight different cities. In the diagram below, each point represents a city and each line segment connecting two points represents a flight connecting two cities.



A traveler in City A has enough frequent flyer points to fly 10 free legs. (A leg is defined to be a non-stop flight from one city to another without stopping at another city along the way.) She decides to go to the airport and randomly choose one of the three available cities. She continues choosing flights in this way until she runs out of frequent flyer points.

- (A) For each of the eight cities, determine the probability that she ends up in that city at the end of her second leg.
- (B) What is the probability that she finishes her trip (after all ten legs) back home in City A?
- (C) Suppose instead that she wants to guarantee that she can get home to A at the end of her trip. How many legs can she choose randomly before she has to plan a route back home? Verify that this strategy would work.

5. Find all the real values of x for which there is a triangle with sides A , B , and C when $A = x^2$, $B = 2x + 1$, and $C = x + 2$.

6. Show that $(\sin 40^\circ)(\sin 80^\circ)(\sin 20^\circ) = \frac{\sqrt{3}}{8}$ exactly. Your work on this question needs to be more than writing my calculator said this was true.

7. The square below is a multiplicative magic square where the product of each row, column, and diagonal is the same. All the entries in the square are positive integers.

50	A	B
C	D	E
F	G	2

- a) What are all the possible values of D?
- b) What are all the possible values of E?
8. The equation $10x^3 + 5ax^2 + bx + a = 0$ has three distinct positive roots, c , d , and e . Suppose $\log_3 c + \log_3 d + \log_3 e = 4$. What is the value of a ?