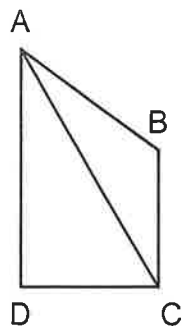


# 2016

## MARSHALL UNIVERSITY MATHEMATICS COMPETITION

1. Given that  $1 + \frac{1}{4} + \frac{1}{9} + \frac{1}{16} + \dots + \frac{1}{n^2} + \dots = \frac{\pi^2}{6}$ ,  
what is the sum  $1 + \frac{1}{9} + \frac{1}{25} + \frac{1}{49} + \dots + \frac{1}{(2n+1)^2} + \dots$ ?
2. The equation  $y = \sqrt{1-x^2}$  defines an upper semicircle of radius 1.
- a. Use geometry (not calculus) to find (and prove) the area under the circle from  $x = -\frac{1}{2}$  to  $x = \frac{\sqrt{3}}{2}$ .
- b. Use geometry (not calculus) to find (and prove) the area under the circle from  $x = \frac{\sqrt{2}}{2}$  to  $x = \frac{\sqrt{3}}{2}$ .
3. A car travels along different paths from A to C at a constant speed  $v$ . It takes 30 minutes to travel AC. It takes 35 minutes to travel AB and BC combined. It takes 40 minutes to travel AD and DC combined. BC and DC are each 10 miles long. If segments DC and BC are perpendicular, solve for the speed of the car  $v$  in miles per hour.



4. A Marshall University math club, the Nerd Herd, has scheduled a train trip for its members. During the trip, members will have access to 36 games and puzzles. The person who earns the most points by winning games and solving puzzles will receive 5 free lunches at a local restaurant, Fat Patty's. When other mathematics students who are not members of the club learned of the trip, they asked if they could come too. The cost of renting the use of a train car for the trip is \$480. In addition to the fare to cover the train car rental, each participant must pay \$20 for lunch and refreshments during the trip. When 5 mathematics students who are not members of the club decide to come on the trip, the fare for all the participants decreases by \$4.80.
- How many members of the club joined the trip?
  - What was the fare?
  - What was the total cost for each participant?
5. Six friends come to Elizabeth's birthday party. She has a square cake with dimensions 14 inches by 14 inches by 3 inches. The cake is iced on all four sides and the top. All seven people at the party love icing. Elizabeth plans to cut the cake so that everyone receives the same amount of icing and the same amount of cake.
- How many square inches of icing should each party goer receive?
  - How many cubic inches of cake should each receive?
  - How would you cut the cake?
6. Suppose that an inherited genetic disorder has two forms: an aggressive form and a passive form. Among all individuals with this disorder, 50% have the aggressive form and 50% have the passive form. If exactly one of a child's parents has the aggressive form of the disorder and the other is healthy, then there is a 60% chance that the child will have the disorder. However, if exactly one of a child's parents has the passive form of the disorder and the other is healthy, then there is only a 20% chance that the child will have the disorder.

Now suppose that Jane and John are married, and that Jane is healthy and John has the disorder, but it is unknown whether the form is aggressive or passive.

- If Jane and John are planning to have their first child, what is the probability that this child will have the disorder?
- Now suppose that Jane and John have had their first child, and that this child has the disorder. If Jane and John decide to have a second child, what is the probability that this second child will have the disorder?

7. The following procedure produces a sequence of numbers called a mathematical black hole:
- 1) Choose any number.
  - 2) Add 12 to it.
  - 3) Multiply the result by 2.
  - 4) Subtract 4.
  - 5) Divide by 4.
  - 6) Subtract half of the number you started with.
- a. What sequence of numbers will be generated if you follow the procedure by beginning with 27?
  - b. What sequence of numbers will be generated if you follow the procedure by beginning with  $\pi$ ? Use exact values.
  - c. What sequence of numbers will be generated if you follow the procedure by beginning with the complex number  $1 + 2i$ ?
  - d. Since in science a black hole is a place where nothing can escape being drawn in, provide a mathematical explanation for why this procedure produces a black hole.
8. There are 8 children at a day care center. For convenience, you can imagine that the children are arbitrarily numbered 1 through 8.
- a. In how many ways can their teacher split them into a group of 4 for finger painting and another group of 4 for modeling clay?
  - b. In how many ways can their teacher split them into two groups of 4 if both groups will be engaging in the same activity?
  - c. In how many ways can their teacher split them into three groups of sizes 4, 2, and 2 if all three groups will be engaging in the same activity?
  - d. Now imagine that there are 30 children instead. Compose a formula for determining the number of ways to split them into three groups of sizes 10, 10, and 10 if all three groups will be engaging in the same activity.