Marshall University Math Colloquium

October 8, 2004

"From Chaos to Stability: Dynamic Equations Parameterized by Time Scales"

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Abstract

Consider the logistic initial value problems

$$x' = 4x\left(\frac{3}{4} - x\right), x(0) = x_0 \quad \text{and} \quad \Delta x = 4x\left(\frac{3}{4} - x\right), x(0) = x_0.$$

For this differential equation, $x(t) = 3/4$ is a stable equilibrium. In forwards time (i.e., as $t \to +\infty$), for x_0 (0,1), all trajectories tend towards 3/4. Finding a solution of the difference equation is equivalent to iterating the function

$$x_{n+1} = 4x_n\left(\frac{3}{4} - x_n\right) + x_n = 4x_n(1 - x_n).$$

Orbits in [0,1] are chaotic except for countably many periodic and pre-periodic orbits. Our long-term goal is to try to understand the differences in behavior between solutions of differential and difference equations as "limits" and "bifurcations" over the underlying domains of the solutions. We use the theory of time scales, developed by S. Hilger in 1988, to do this. The set of closed subsets of $\mathbb{R}_+ = [0,\infty)$, $\mathrm{CL}(\mathbb{R}_+)$, is a parameter space for the corresponding dynamic equations

$$x^{\Delta} = 4x\left(\frac{3}{4} - x\right), x(0) = x_0.$$

The time scales for the forward solutions of the differential equation and the difference equation are $\mathbb{R}_+$ and $\mathbb{Z}_+$, respectively. This is joint work with E. R. Duke, K. J. Hall, and B. A. Lawrence.