The topology of chain selected complexes of a poset

JiYoon Jung

For each composition \( \vec{c} \) we show that the order complex of the poset of pointed set partitions \( \Pi^*_{\vec{c}} \) is a wedge of \( \beta(\vec{c}) \) spheres of the same dimensions, where \( \beta(\vec{c}) \) is the number of permutations with descent composition \( \vec{c} \). Furthermore, the action of the symmetric group on the top homology is isomorphic to the Specht module \( S^B \) where \( B \) is a border strip associated to the composition \( \vec{c} \). We also study the filter of pointed set partitions generated by knapsack integer partitions and show the analogous results on homotopy type and action on the top homology.

Let \( \Lambda \) be a sub semi-group of the natural numbers \( \mathbb{N} \). I am interested in filters of the partition lattice consisting of partitions where every block size belongs to \( \Lambda \). Since \( \Lambda \) is closed under addition, these collections of partitions do form a filter in the partition lattice. In the case that \( \Lambda \) is generated by \( \langle a, d \rangle \) where \( a \) and \( d \) are relatively prime, I conjecture that the action of the symmetric group on the top homology group of the associated order complexes is a direct sum of Specht modules. Precisely, I conjecture that \( \tilde{H}_k(\Delta) \) is isomorphic to the direct sum of Specht modules \( \bigoplus_{\vec{c}} S^{\beta(\vec{c})} \) where \( \vec{c} = (c_1, c_2, \ldots, c_{k+2}) \) ranges over all compositions of \( n \) satisfying \( c_i = a + m_i \cdot d \) for \( 0 \leq m_i < a \).

This is joint work with Richard Ehrenborg.