The topology of chain selected complexes of a poset

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For each composition $\vec{c}$ we show that the order complex of the poset of pointed set partitions $\Pi^{\bullet}_{\vec{c}}$ is a wedge of $\beta(\vec{c})$ spheres of the same dimensions, where $\beta(\vec{c})$ is the number of permutations with descent composition $\vec{c}$. Furthermore, the action of the symmetric group on the top homology is isomorphic to the Specht module S^B where B is a border strip associated to the composition $\vec{c}$. We also study the filter of pointed set partitions generated by knapsack integer partitions and show the analogous results on homotopy type and action on the top homology.

Let Λ be a sub semi-group of the natural numbers $\mathbb{N}$. I am interested in filters of the partition lattice consisting of partitions where every block size belongs to Λ. Since Λ is closed under addition, these collections of partitions do form a filter in the partition lattice. In the case that Λ is generated by $\langle a,\ d\rangle$ where a and d are relatively prime, I conjecture that the action of the symmetric group on the top homology group of the associated order complexes is a direct sum of Specht modules. Precisely, I conjecture that $\widetilde{H}_k(\Delta)$ is isomorphic to the direct sum of Specht modules $\bigoplus_{\vec{c}} S^{\beta(\vec{c})}$ where $\vec{c} = (c_1, c_2, \ldots, c_{k+2})$ ranges over all compositions of n satisfying $c_i = a + m_i \cdot d$ for $0 \leq m_i < a$.

This is joint work with Richard Ehrenborg.