30th Cumberland Conference on Combinatorics, Graph Theory, and Computing

Marshall University

May 19–20, 2018

Supported by a grant from the
U.S. National Science Foundation (DMS-1822390)
and by Marshall University

Conference Photograph

May 19, 2018

Conference Website

https://www.marshall.edu/math/cumberland2018/

Schedule of Talks

Saturday, May 19		
8:00	**Registration** Outside SH 154	
8:30	**Colbourn** (P1) SH 154	
9:30	**Rolek** (C27) SH 154	**Dougherty*** (C13) SH 335
9:55	**Ellingham** (C14) SH 154	**Pllaha***(C24) SH 335
10:20	**Coffee** Outside SH 154	
10:45		**Bona** (C3) SH 335
11:10	**Horacek***(C18) SH 154	**Smith** (C30) SH 335
11:35	**Skogman** (C29) SH 154	**Stucky***(C31) SH 335
12:00	**Lunch break**	
1:45	**Warrington** (P3) SH 154	
2:45	**Robinson** (C26) SH 154	**Yatauro** (C33) SH 335
3:10	**Owsley***(C23) SH 154	**Bowie** (C5) SH 335
3:35	**Coffee** Outside SH 154	
4:00	**Asplund** (C1) SH 154	**May** (C21) SH 335
4:25	**Busch** (C7) SH 154	**Feller***(C15) SH 335
4:50	**Rusnak** (C28) SH 154	**Tam***(C32) SH 335
5:15	**Liu** (C20) SH 154	

Sunday, May 20		
8:00	**Registration** Outside SH 154	
8:30	**Young** (P4) SH 154	
9:30	**DeBiasio** (C12) SH 154	**Braun** (C6) SH 335
9:55	**Krueger***(C19) SH 154	**Nastase** (C22) SH 335
10:20	**Coffee** Outside SH 154	
10:45	**Chen***(C9) SH 154	**Chatham** (C8) SH 335
11:10	**Zhang***(C34) SH 154	**Davis** (C11) SH 335
11:35	**Bosse***(C4) SH 154	**Goddard** (C16) SH 335
12:00	**Box lunch** (Provided) Outside SH 154	
1:00	**Hersh** (P2) SH 154	
2:00	**McKenney** (C35) SH 154	**Choi** (C10) SH 335
2:25	**Por** (C25) SH 154	

An asterisk (*) indicates a student speaker.

Plenary Talks

P1 Asymptotic and constructive bounds for sequence covering arrays

Charlie Colbourn, Arizona State University (Charles.Colbourn@asu.edu)

A sequence covering array for parameters n, t, and v is a set of n permutations of v letters so that every permutation of every t of the v letters appears in the specified order in at least one of the n permutations. The motivation for sequence covering arrays arises in event sequence testing. Suppose that a process involves a sequence of k tasks. The operator may, unfortunately, fail to do the tasks in the correct sequence. When this happens, errors may occur. Often, errors can be attributed to the (improper) ordering of a small subset of tasks. When each permutation of a sequence covering array is used in turn to specify a task order, every potential ordering of each t tasks will be tried, and hence all errors found that can be attributed to t or fewer tasks.

Efficient algorithms to construct sequence covering arrays are of interest. We explore two asymptotic bounds, one from an elementary probabilistic argument and one via the Lovàsz Local Lemma. Using the conditional expectation method for the first, and the Moser–Tardos resampling method for the second, we describe constructive methods that are easily implemented and prove to be effective for relatively large parameters. Finally we adapt the random selection to use oversampling, and discuss both the asymptotic and the algorithmic consequences.

This is joint work with Yun Li and Lijun Ji (Suzhou, China).

P2 Posets arising as 1-skeleta of simple polytopes, diameter bounds on polytopes, and poset topology

Patricia Hersh, North Carolina State University (plhersh@ncsu.edu)

Given a polytope P and generic linear functional c, one obtains a directed graph $G(P,c)$ by taking the 1-skeleton of P and orienting each edge $e(u,v)$ from u to v for $c(u) < c(v)$. We will discuss the question of finding sufficient conditions on P and c so that $G(P,c)$ will not have any directed paths which revisit a face of P after departing from it. This is equivalent to the question of finding conditions on P and c under which the simplex method for linear programming will be efficient under all choices of pivot rules. Conditions are given which provably yield a corollary of the desired nonrevisiting property. One of the proposed conditions is that $G(P,c)$ be the Hasse diagram of a partially ordered set, which is equivalent to requiring nonrevisiting of 1-dimensional faces. This opens the door to the usage of poset-theoretic techniques. This also leads to a result for simple polytopes in which $G(P,c)$ is the Hasse diagram of a lattice L that the order complex of each open interval in L is homotopy equivalent to a ball or a sphere, with applications to weak Bruhat order, the Tamari lattice and Cambrian lattices. We will tell this story, providing background and history along the way.

P3 Quasisymmetric functions in algebraic combinatorics

Greg Warrington, University of Vermont (gswarrin@uvm.edu)

Quasisymmetric functions are ubiquitous in algebraic combinatorics. In addition to arising naturally in many enumerative problems, they also provide a powerful connection to representation theory and Schur functions. In this talk we'll survey this landscape in addition to describing some recent results.

P4 Polychromatic colorings of hypercubes and integers

Michael Young, Iowa State University (myoung@iastate.edu)

Given a graph G which is a subgraph of the n-dimensional hypercube Q_n, an edge coloring of Q_n with $r \geq 2$ colors such that every copy of G contains every color is called G-polychromatic. Originally introduced by Alon, Krech and Szabó in 2007 as a way to prove bounds for Turán type problems on the hypercube, polychromatic colorings have proven to be worthy of study in their own right. This talk will survey what is currently known about polychromatic colorings and introduce some open questions. In addition, there are some natural generalizations and variations of the problem that will also be discussed. One generalization will be polychromatic colorings of the integers, which we use to prove a conjecture of Newman.

Contributed Talks

An asterisk (*) indicates a student speaker.

C1 2-block intersection graphs on triple systems

Speaker: John Asplund, Dalton State College (`jasplund@daltonstate.edu`)

Often when we go to presentations, we see only the end product of one's multi-year project(s). What goes into an investigation and what holes-of-thought do we get stuck on as we progress through our project? In this talk, I'll focus on the Hamiltonicity of 2-block intersection graphs (2-BIG) of a triple system. Simply put, a triple system is a tool for building a certain class of graphs and we are interested in showing which 2-BIGs contain a cycle that spans the entire graph. This talk will chronicle the methods and results on the Hamiltonicity of 2-BIGs of triple systems over the last two years and our future work.

C3 Cyclic permutations avoiding pairs of short patterns

Speaker: Miklos Bona, University of Florida (`bona@ufl.edu`)

Enumerating permutations that consist of a single cycle and avoid certain patterns is a notoriously difficult family of problems, raised first by Richard Stanley in 2011. In this talk, we present exact formulas for two pairs of patterns of length three each.

C4 Multicolor Gallai-Ramsey numbers of odd cycles

Speaker: Christian Bosse*, University of Central Florida (`csbosse@knights.ucf.edu`)

A Gallai-coloring of a complete graph is an edge-coloring such that no triangle is colored with three distinct colors. We study Ramsey-type problems in Gallai-colorings. Given a graph H and an integer $k \geq 1$, the Gallai-Ramsey number $GR_k(H)$ is the least positive integer n such that every Gallai-coloring of K_n with k colors contains a monochromatic copy of H. It turns out that $GR_k(H)$ is more well-behaved than the classical Ramsey number $R_k(H)$. However, finding exact values of $GR_k(H)$ is far from trivial. In this talk, I will present our recent results on Gallai-Ramsey numbers of odd cycles. We prove that $GR_k(C_{2n+1}) = n \cdot 2^k + 1$ for $n \in \{4, 5, 6, 7\}$ and all integers $k \geq 1$. This provides partial evidence for the first four open cases of the Triple Odd Cycle Conjecture of Bondy and Erdős from 1973. We believe the method presented here may be helpful in determining the exact values of $GR_k(C_{2n+1})$ for all $n \geq 8$.

C5 Set-sized parameters

Speaker: Miranda Bowie, University of North Alabama (`mbowie@una.edu`)

For a graph G, the domination number, $\gamma(G)$, is defined to be $min\{|D| : D \subseteq V(G)$ and $|N[v] \cap D| \geq 1$ for each $v \in V(G)\}$. For a graph G, the packing number, $\rho(G)$, is defined to be $max\{|S| : S \subseteq V(G)$ and $|N[v] \cap S| \leq 1$ for each $v \in V(G)\}$. Note that for every vertex $v \in V(G)$ there is a requirement on the number of vertices in a dominating set D which must lie in the closed neighborhood of v. Also, for every vertex $v \in V(G)$ there is a restriction on the number of vertices in a packing set S which can lie within the closed neighborhood of v. We will consider set-sized parameters which extend these requirements and restrictions from individual vertices to collections of vertices.

C6 Matching and independence complexes related to small grids

Speaker: Benjamin Braun, University of Kentucky (`benjamin.braun@uky.edu`)

Co-author: Wesley Hough, University of Wisconsin-Whitewater

The topology of the matching complex for the 2xn grid graph is mysterious. We describe a discrete Morse matching for a family of independence complexes that include these matching complexes. Using this matching, we determine the dimensions of the chain spaces for the resulting Morse complexes and derive bounds on the location of non-trivial homology groups for some of these complexes. Further, we determine the Euler characteristic of these complexes and prove that several of their homology groups are non-zero.

C7 Completions of k-colored graphs avoiding prescribed chordless cycles

Speaker: Arthur Busch, University of Dayton (`art.busch@udayton.edu`)

Co-author: R. Sritharan (University of Dayton, Dept. of Computer Science)

In this talk, we consider a special case of sandwich problems typically referred to as colored completion problems. Given a a properly colored graph G, the goal is to determine if some properly colored supergraph of G exists with a specified property. Sandwich problems are a natural extension of recognition problems, and as such we seek to determine whether the colored completion problem for a particular property can be solved in polynomial time. In this talk we consider the complexity of determining whether a properly colored supergraph of a k-colored graph exists that avoids a given set of chordless cycles.

C8 Reflections on the $n + k$ dragon kings problem

Speaker: Doug Chatham, Morehead State University
(`d.chatham@moreheadstate.edu`)

A *dragon king* is a shogi piece that moves any number of squares vertically or horizontally or one square diagonally but does not move through or jump over other pieces. We construct infinite families of solutions to the *$n + k$ dragon kings problem* of placing k pawns and $n + k$ mutually nonattacking dragon kings on an $n \times n$ board, including solutions symmetric with respect to quarter-turn or half-turn rotations, solutions symmetric with respect to one or two diagonal reflections, and solutions not symmetric with respect to any nontrivial rotation or reflection. We show that an $n + k$ dragon kings problem solution exists whenever $n \geq k + 5$ and that, given some extra conditions, symmetric solutions exist for $n \geq 2k + 5$.

C9 Planar anti-Ramsey numbers of matchings

Speaker: Gang Chen*, Ningxia University (chengang16@yahoo.com)

Co-authors: Yongxin Lan, Nankai University; Zi-Xia Song, University of Central Florida

Given a positive integer n and a planar graph H, let $\mathcal{T}_n(H)$ be the family of all plane triangulations T on n vertices such that T contains a subgraph isomorphic to H. The **planar anti-Ramsey number of** H, denoted $ar_{\mathcal{P}}(n, H)$, is the maximum number of colors in an edge-coloring of a plane triangulation $T \in \mathcal{T}_n(H)$ such that T contains no rainbow copy of H. In this paper we study planar anti-Ramsey numbers of matchings. For all $t \geq 1$, let M_t denote a matching of size t. We prove that for all $t \geq 6$ and $n \geq 3t - 6$, $2n + 3t - 15 \leq ar_{\mathcal{P}}(n, M_t) \leq 2n + 4t - 14$, which significantly improves the existing lower and upper bounds for $ar_{\mathcal{P}}(n, M_t)$. It seems that for each $t \geq 6$, the lower bound we obtained is the exact value of $ar_{\mathcal{P}}(n, M_t)$ for sufficiently large n. This is indeed the case for M_6. We prove that $ar_{\mathcal{P}}(n, M_6) = 2n + 3$ for all $n \geq 30$.

C10 Digit sums of ternary strings and trinomial coefficients

Speaker: Ji Young Choi, Shippensburg University of PA (jychoi@ship.edu)

Let x be a ternary string representing a positive integer m. We define indispensable digits in x so that the digit sum of the ternary string representing $2m$ is 2 times the number of indispensable digits in x. This talk will present that the number of ternary strings of length n with k indispensable digits in terms of trinomial coefficients.

C11 A direct construction of non-transitive dice sets

Speaker: Matt Davis, Muskingum University (mattd@muskingum.edu)

Co-author: Levi Angel, Muskingum University

Given a set of dice numbered in a non-standard way, it is natural to examine the relation on the dice where $A > B$ exactly if A has a greater than 50% chance of rolling higher than B. It is easy to number the dice in a way that makes this relation non-transitive. In this talk, we describe a construction that allows us to create a set of dice that realize an arbitrary relation. This construction is non-inductive and is generally more efficient than other known constructions.

C12 Monochromatic paths/cycles in bipartite graphs I

Speaker: Louis DeBiasio, Miami University (debiasld@miamioh.edu)

Co-authors: Andras Gyarfas, Bob Krueger, Gabor Sarkozy, Miklos Ruszinko

Given an r-coloring of the edges of a balanced bipartite graph, what is the length of a longest monochromatic path/cycle? We discuss what is known in the case of an r-colored complete bipartite graph.

C13 An improved recursive construction for perfect hash families of large strength

Speaker: Ryan Dougherty*, Arizona State University (`ryan.dougherty@asu.edu`)

Co-author: Charles Colbourn, Arizona State University

Hash families (HFs) are combinatorial objects that have applications in the construction of covering arrays, separating systems, IPP codes, and more. Determination of the smallest size of HFs given the number of columns, symbols, and strength is of great theoretical and practical interest. We present a new recursive construction for perfect hash families of large strength that improves the best known upper bound for many parameters.

C14 The Chvátal-Erdős condition for prism-hamiltonicity

Speaker: Mark Ellingham, Vanderbilt University
(`mark.ellingham@vanderbilt.edu`)

Co-author: Pouria Salehi Nowbandegani

A graph G is *prism-hamiltonian* if the prism $G \square K_2$ is hamiltonian. In general, being prism-hamiltonian is weaker than having a hamilton path, but stronger than having a spanning closed 2-walk (a walk using every vertex at most twice). Chvátal and Erdős showed that a graph G has a hamilton cycle if $\alpha(G) \leq \kappa(G)$, where $\alpha(G)$ is the independence number and $\kappa(G)$ is the connectivity. As corollaries of this, G has a hamilton path if $\alpha(G) \leq \kappa(G) + 1$, and a spanning closed 2-walk if $\alpha(G) \leq 2\kappa(G)$; these results are best possible. We show that the same condition that guarantees a spanning closed 2-walk, $\alpha(G) \leq 2\kappa(G)$, also guarantees the stronger conclusion of prism-hamiltonicity, answering a question of West.

C15 Ramsey numbers from tilings of T-tetrominos

Speaker: Emily Feller*, University of Dallas (`efeller@udallas.edu`)

Co-author: Elizabeth Derdeyn

The ordinary van der Waerden Number $W(k, l)$ is the least positive integer W such that any k-coloring of the first W positive integers contains a monochromatic arithmetic progression of length l. We consider tilings of rectangles by T-tetrominos, and ask when arithmetic progressions (AP) of tiles (equally-spaced and in the same orientation) must occur. Let the notation $(M, N) \to l$ mean that any tiling of an $M \times N$ rectangle by T-tetrominos must contain an l-term AP. We will present all known pairs (M, N) such that $(M, N) \to l$ for $l \in \{1, 2, 3, 4\}$, and the methods used to identify these pairs. We characterize all (M, N) such that $(M, N) \not\to 3$, we prove that $(4, 4N) \to l \iff N \geq W(2, l)$, and present evidence that in the tiling scenario, we may not have monotonicity: That is, $(M, N_1) \to l$ and $N_2 > N_1$ might not imply $(M, N_2) \to l$.

C16 The matcher game played in graphs

Speaker: Wayne Goddard, Clemson University (goddard@clemson.edu)

We introduce a game played on a graph by two players, named Max and Min. Each round, first Max chooses a vertex u that has at least one unchosen neighbor and then Min chooses a neighbor of u. This process eventually produces a maximal matching. The matcher number is the number of edges chosen with optimal play, where Max tries to maximize this quantity and Min tries to minimize it. We show that the matcher number is always at least 2/3 the matching number. But for bipartite graphs the two parameters are equal. We also provide some results on graphs of large odd girth and on dense graphs.

C18 List star edge coloring of sparse graphs

Speaker: Katie Horacek*, West Virginia University (khoracek@math.wvu.edu)

Co-authors: Jiaoo Li, West Virginia University; Rong Luo, West Virginia University

A *star k-edge coloring* is a proper k-edge-coloring such that there is no cycle or path of length 4 that contains only 2 colors. The star chromatic index of a graph G is the smallest integer k such that G admits a star k-edge-coloring. Bezegova et al. and Deng et al. (independently) proved that the star chromatic index of a tree with maximum degree Δ is at most $\frac{3\Delta}{2}$, and the bound is tight. Our result extends this idea to list star chromatic index of graphs with maximum average degree $mad(G) < \frac{8}{3}$: specifically, for a graph G with $mad(G) = \frac{8}{3} - \varepsilon$, $ch'_{st}(G) \leq \frac{3\Delta}{2} + c(\varepsilon)$, where $c(\varepsilon)$ is a constant dependent only on ε. This result implies bounds for planar graphs of large girth.

C19 Monochromatic paths/cycles in bipartite graphs II

Speaker: Bob Krueger*, Miami University (kruegera@miamioh.edu)

Co-author: Loudis DeBiasio

Given an r-coloring of the edges of a balanced bipartite graph, what is the length of a longest monochromatic path/cycle? We discuss what is known in the case of a 2-coloredd bipartite graph with a given minimum degree.

C20 The tightness of the Kesten-Stigum reconstruction bound on regular trees

Speaker: Wenjian Liu, Queensborough Community College, CUNY (wjliu@qcc.cuny.edu)

BACKGROUND: Determining the reconstruction threshold of a broadcast models on d-ary regular tree, as the interdisciplinary subject, has attracted more and more attention from probabilists, statistical physicists, biologists, etc.

OBJECTIVE: Consider a $2q$-state symmetric transition matrix as the noisy communication channel on each edge of a regular d-ary tree. Suppose there are two categories, one of which contains exactly q states, and 3 transition probabilities: remain in the same state, mutate to other state but remain in the same category, and mutate to the other category. Consider all the symbols received at the vertices of the nth generation. Does this leaves-configuration contain a non-vanishing information on the letter transmitted by the root, as n goes to infinity? By means of a refined analysis of moment recursion on a weighted version of the magnetization, concentration investigation, and large degree asymptotics, we construct a nonlinear second-order dynamical system and show that the KestenStigum reconstruction bound is not tight when $q \geq 4$. On the other side, when $q = 2$, that is, Kimura Model of DNA evolution, the interactions of nodes on tree become weaker as d increases. This allows us to utilize the Gaussian approximation. Therefore, we explore stability of the fixed point of Gaussian approximation function in order to verify the tightness of Kesten-Stigum reconstruction bound.

APPLICATION: The reconstruction problem is concerned essentially with a tradeoff between noise and duplication in a tree communication network; phylogenetic reconstruction is a major task of systematic biology; reconstruction thresholds on trees are believed to determine the dynamic phase transitions in many constraint satisfaction problems including random K-SAT and random colorings on random graphs; the reconstruction threshold is also believed to play an important role in the efficiency of the Glauber dynamics on trees and random graphs.

C21 Progress towards counting knot mosaics with graph colorings and alternating sign matrices

Speaker: Rus May, Morehead State University (`r.may@moreheadstate.edu`)

Knot mosaics are arrangements of tiles with strands of rope into a grid. Exact enumeration of knot mosaics of general size has been an open problem ever since 2010 in a seminal paper by Lomonaco and Kauffman. Knot mosaics can be represented as two-colorings of a lattice. They are also related to alternating sign matrices. We discuss these relations and progress towards obtaining an exact count of knot mosaics.

C22 Partitions of a finite vector space into subspaces of two different dimensions

Speaker: Esmeralda Nastase, Xavier University (`nastasee@xavier.edu`)

Co-author: Papa Sissokho, Illinois State University

Let q be a prime power and let n, t, s, and r be integers such that $r \equiv n \pmod{t}$, $0 \leq r < t$, and $s < t < n$. Let $V(n,q)$ be the finite vector space of dimension n over GF(q). A (t,s)*-partition* of $V(n,q)$ is a collection $\mathcal{P}$ of subspaces of $V(n,q)$ whose dimensions are from the set $\{t,s\}$ and such that each nonzero vector in $V(n,q)$ is contained in exactly one of the subspaces in $\mathcal{P}$. Let $\mathcal{F}_q(n,t,s)$ denote the set of all (t,s)-partition of $V(n,q)$. For $\mathcal{F}_q(n,t,s) \neq \emptyset$, we let $\mu_q(n,t,s)$ be the maximum number of t-dimensional subspaces over all subspace partitions in $\mathcal{F}_q(n,t,s)$. We determine $\mu_q(n,t,s)$ for various values of $s \geq 2$. In particular, if $t > (q^r-1)/(q-1)$, we determine the number $\mu_q(n,t,s)$ whenever $s = 2$, and when s divides r or $t+r$. This generalizes a recent result by the authors who determined the *maximum size of a partial t-spread* of $V(n,q)$, i.e., $\mu_q(n,t,1)$, for $t > (q^r-1)/(q-1)$.

C23 On primitive cycle decompositions of K_n and $K_n - I$

Speaker: Kira Owsley*, Marshall University (`owsley4@marshall.edu`)

Let G be a connected graph and $\mathcal{D}$ be a decomposition of G into subgraphs with no isolated vertices. A subset $\mathcal{D}' \subseteq \mathcal{D}$ is a **subsystem** of $\mathcal{D}$ if $\mathcal{D}'$ is a decomposition of an induced subgraph of G. We say that $\mathcal{D}$ is primitive if the only subsystems of $\mathcal{D}$ are $\mathcal{D}$ and $\varnothing$. In this talk we will relay recent results in the existence of primitive cycle systems for complete graphs and complete graphs minus a 1-factor. This is continuing work with John Asplund, Venkata Dinvahi, and Michael W. Schroeder.

C24 Symplectic isometries of quantum stabilizer codes

Speaker: Tefjol Pllaha*, University of Kentucky (`tefjol.pllaha@uky.edu`)

Quantum stabilizer codes form the most important class of quantum error-correcting codes. In this talk we define quantum stabilizer codes over Frobenius rings and establish a correspondence with self-orthogonal submodules CR^{2n} under a certain bilinear form, called stabilizer codes. The equivalence of stabilizer codes is discussed via symplectic isometries. We study the isometry groups of stabilizer codes and show that they can be as different as possible.

C25 k-monotone interpolation

Speaker: Attila Por, Western Kentucky University (`attila.por@wku.edu`)

Co-authors: Martin Balko, Geza Toth, Pavel Valtr

A classical result of Erdős and Szekeres state that in a sequence of real numbers of length n^2+1 one can find a monotone subsequence of length $n+1$; and in a set of $\binom{2n-4}{n-2}+1$ points in the plane one can find a subset of size n that is either a "cup" (convex) or a "cap". Alternatively one can call a set of points a cup, if there exists a function $f(x)$ such that the points lie on the graph of f, and the function is convex, that is the first derivative is increasing and the second derivative is positive. Extending this definition a set of points in the plane is k-monotone-interpolable if they lie on the graph of a function f or $-f$ where the $(k-2)$th derivative of f is convex. Now a cap is 2-monotone interpolable, and an ancreasing sequence is 1-monotone interpolable. The classical results are the cases $k = 1$ and $k = 2$ for the following Ramsey-type statement: For every n, k positive integers, there exists $N = N(n, k)$ such that any set of N points in the plane with different x-coordinates has a subset of size n that is k-monotone interpolable.

The case $k = 3$ was proved by Cibulka, Matousek and Patak. We show that it is true for any $k > 3$.

This result is non-trivial, since the statements: "A sequence is increasing if every two elements are increasing"; and "A set of points is a cup if every triple is a cup" do not generalize. It was show by [CMP] that for every $n > 4$ there exists a set of $n+1$ points in the plane such that every proper subset is three-monotone interpolable but the whole set is not.

C26 Counting cycle decompositions of labeled 2-biregular digraphs

Speaker: Robert W. Robinson, University of Georgia (rwr@uga.edu)

By 2-biregular is meant that every vertex has out-degree 2 and in-degree 2. A cycle decomposition of such a digraph consists of a set of edge-disjoint directed cycles such that every vertex lies on exactly two cycles in the set. We count d_n, the total number of cycle decompositions summed over all labeled 2-biregular digraphs of order n.

We also find f_n, the total number of factorizations into edge-disjoint 1-biregular digraphs, summed over all labeled 2-biregular digraphs of order n.

Exponential generating functions and inclusion-exclusion are used to determine d_n and f_n.

C27 The extremal function for $K_9^{=}$ minors

Speaker: Martin Rolek, College of William & Mary (msrolek@wm.edu)

We prove the extremal function for $K_9^{=}$ minors, where $K_9^{=}$ denotes the complete graph K_9 with two edges removed. In particular, we show that any graph with n vertices and at least $6n - 20$ edges either contains a $K_9^{=}$ minor or is isomorphic to a graph obtained from disjoint copies of K_8 and $K_{2,2,2,2,2}$ by identifying cliques of size 5. We utilize computer assistance to prove several of our lemmas, and we provide a discussion of this use in our talk.

C28 Matrix-tree theorems for integer matrices via oriented hypergraphs

Speaker: Lucas J. Rusnak, Texas State University (lr27@txstate.edu)

Co-authors: W. Grilliette, E. Robinson, P. Shroff

An oriented hypergraph provides a combinatorial way to study integer matrices through its locally signed graphic sub-structure. We will examine the naturality of integer matrix versions of the Matrix-tree Theorem, Tutte's transpedance theorem, and Sach's coefficient theorem along with their "line graph" analogs.

The incidence theoretic results are a result of comparing combinatorial categories and exhibiting a natural functor between the category of incidence structures and quivers. Incidence matrices and bipartite representation graphs arise as right and left adjoints of this functor, and familiar graph theoretic concepts are reclaimed by quotients of the non-trivial generator and by adjusting the monotonicity of incidence-preserving maps.

The techniques introduced both specialize to provide determinant and permanent bounds on minors of integer matrices, and generalize to functor categories.

C29 Edge weighted covering graphs and Ramanujan graphs

Speaker: Howard Skogman, SUNY Brockport (hskogman@brockport.edu)

Co-author: Marvin Minei

We define an edge weighted covering graph and use it to create Ramanujan graphs. While the examples we give are high degree examples, the construction gives insight into the general problem of creating Ramanujan graphs as covering graphs.

C30 Sorting permutations with $t + 1$ passes

Speaker: Rebecca Smith, SUNY Brockport (rnsmith@brockport.edu)

Co-authors: Toufik Mansour (University of Haifa), Howard Skogman (SUNY Brockport)

Knuth showed that a permutation π can be sorted by a stack (meaning that by applying push and pop operations to the sequence of entries $\pi(1), \pi(2), \ldots, \pi(n)$ we can output the sequence 12...n (the identity permutation) if and only if p avoids the permutation pattern 231. Note that π is said to avoid 231 if and only if there do not exist three indices $1 \leq i_1 < i_2 < i_3 \leq n$ such that $\pi(i_1), \pi(i_2), \pi(i_3)$ are in the same relative order as 231.

We consider the number of passes a permutation needs to take through a stack if we only pop the appropriate output values and start over with the remaining entries. One result we obtain is that the class of permutations that require exactly $t + 1$ passes to be sorted (where remaining non-sorted entries are passed through the stack in their original order) is in bijection with a collection of integer sequences enumerated by Susan Parker.

C31 A mystery poset on "half" of the Catalan objects

Speaker: Eric Stucky*, University of Minnesota (stuck127@umn.edu)

Catalan numbers count "Catalan objects", some of the most well-studied objects in enumerative combinatorics. They have several natural q-analogues, including one proposed by MacMahon in his 1913 investigation of the major index. In 1994, Haiman observed that while MacMahon's polynomial is not unimodal, its even and odd parts are! Haiman's (rather difficult) proof of this fact suggests that the two parts of MacMahon's polynomial may naturally occur as the rank generating functions for some as-of-yet mysterious posets on "even" and "odd" Catalan objects.

In this talk, we describe some initial success with designing appropriate posets, including a simple interpretation of even and odd Catalan objects (and their analogues in rational Schröder generality).

C32 Finitarization of infinite matroids

Speaker: Patrick Tam*, UC Davis (patrick.c.tam@gmail.com)

We consider the open problem concerning whether every nearly finitary matroid is k-nearly finitary for some k. We introduce a notion of finitarization spectrum. We compute this finitarization spectrum in a few examples. In particular, we show that this spectrum can be larger than any fixed finite size.

C33 Stability theorems for the binding number and tenacity of a graph

Speaker: Michael Yatauro, Penn State, Brandywine Campus (`mry3@psu.edu`)

Let G be a finite, simple graph. For a subset S of $V(G)$, let N(S) be the neighbor set of S, let $w(G-S)$ be the number of components of the induced subgraph $< G-S >$, and let $m(G-S)$ be the order of a largest component of the induced subgraph $< G-S >$. The binding number of G, introduced by Woodall and denoted by $\text{bind}(G)$, is the minimum value obtained by the ratio $|N(S)|/|S|$ when considering all possible subsets S of $V(G)$ and the tenacity of G, denoted $T(G)$, is the minimum value obtained by the ratio $(|S|+m(G-S))/w(G-S)$ when considering all possible subsets S of $V(G)$. If $\text{bind}(G)$ is at least b, we say G is b-binding and if $T(G)$ is at least t, we say G is t-tenacious. We present a theorem which provides a number k so that if $\deg(x)+\deg(y)$ is at least k for some non-adjacent vertices x, y of G, then G is b-binding if and only if $G+xy$ is b-binding. We then provide a similar theorem for t-tenacious. We also demonstrate that each value of k is best possible.

C34 Multicolor Gallai-Ramsey numbers of even cycles and paths

Speaker: Jingmei Zhang*, University of Central Florida (`jmzhang@knights.ucf.edu`)

Co-author: Zi-Xia Song (University of Central Florida)

A *Gallai coloring* is a coloring of the edges of a complete graph without rainbow triangles. Given an integer $k \geq 1$ and graphs $H_1, H_2, \ldots, H_k$, the Gallai-Ramsey number $GR(H_1, H_2, \ldots, H_k)$ is the least integer n such that every Gallai coloring of the complete graph K_n with k colors contains a monochromatic copy of H_i in color i for some $i \in \{1, 2, \ldots, k\}$. When $H = H_1 = \cdots = H_k$, we simply write $GR_k(H)$. We study Gallai-Ramsey numbers of even cycles and paths. For all $n \geq 3$ and $k \geq 1$, let $G_i = P_{2i+3}$ be a path on $2i+3$ vertices for all $i \in \{0, 1, \ldots, n-2\}$ and $G_{n-1} \in \{C_{2n}, P_{2n+1}\}$. Let $i_j \in \{0, 1, \ldots, n-1\}$ for all $j \in \{1, 2, \ldots, k\}$ with $i_1 \geq i_2 \geq \cdots \geq i_k$. Song recently conjectured that $GR(G_{i_1}, G_{i_2}, \ldots, G_{i_k}) = 3 + \min\{i_1, n^* - 2\} + \sum_{j=1}^{k} i_j$, where $n^* = n$ when $G_{i_1} \neq P_{2n+1}$ and $n^* = n+1$ when $G_{i_1} = P_{2n+1}$. The truth of this conjecture implies that $GR_k(C_{2n}) = GR_k(P_{2n}) = (n-1)k + n + 1$ for all $n \geq 3$ and $k \geq 1$, and $GR_k(P_{2n+1}) = (n-1)k + n + 2$ for all $n \geq 1$ and $k \geq 1$. In this talk, I will present that the aforementioned conjecture holds for $n \in \{3, 4, 5\}$ and all $k \geq 1$. Our proof relies only on Gallai's result and the classical Ramsey numbers $R(H_1, H_2)$, where $H_1, H_2 \in \{C_{10}, C_8, C_6, P_9, P_7, P_5, P_3\}$. We believe the recoloring method we developed here will be very useful for solving subsequent cases, and perhaps the conjecture.

C35 Asymptotic density of monochromatic subgraphs

Speaker: Paul McKenney, Miami University (`mckennp2@miamioh.edu`)

Co-author: Louis DeBiasio (Miami University)

Ramsey's theorem implies that every countably infinite graph can be found as a monochromatic subgraph in any 2-coloring of the complete graph on the natural numbers. A natural question to ask is: how dense (in the natural numbers) can we make this monochromatic subgraph? I will discuss some partial answers for various types of subgraphs, including paths, locally finite bipartite graphs, and others.

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