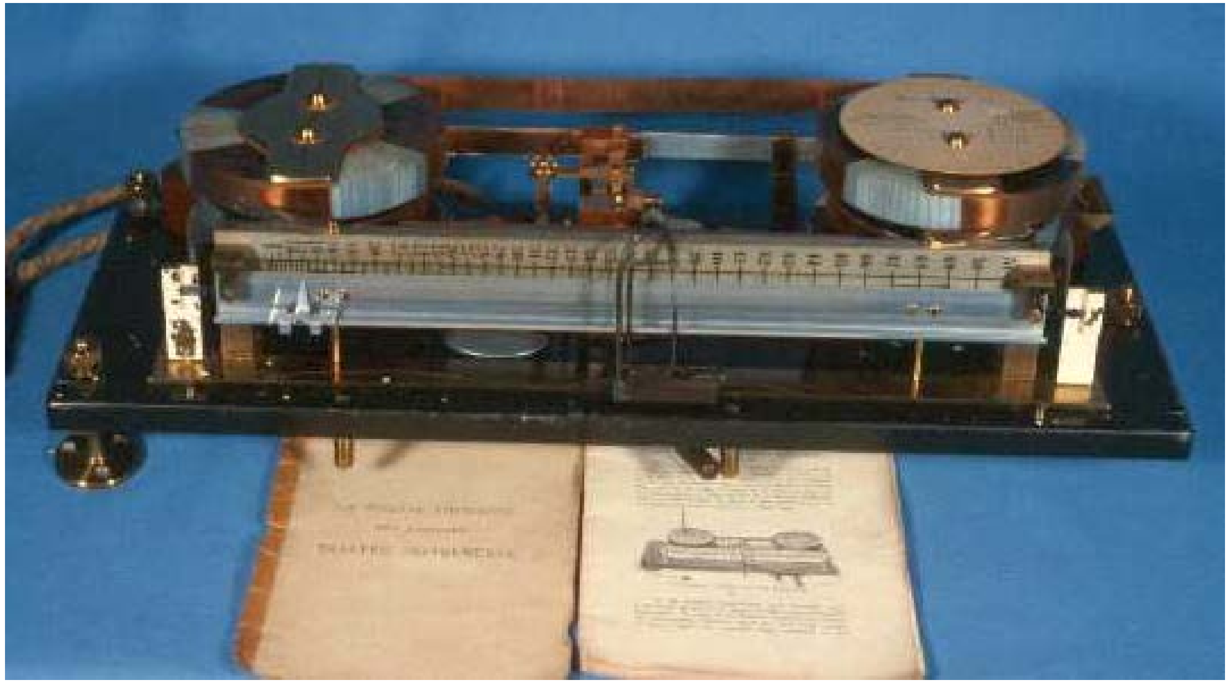
Compact 3-D Printed Kelvin Current Balance

Thomas E Wilson, Department of Physics, Marshall University, Huntington, WV

wilsont@marshall.edu

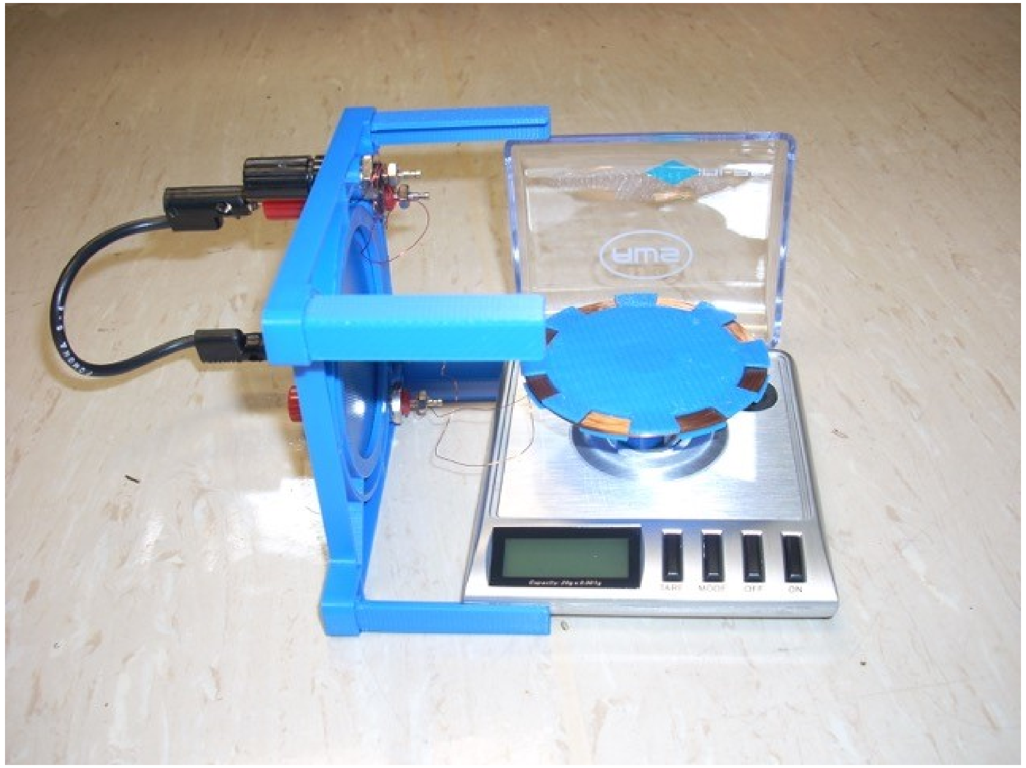
Current Balance of Lord Kelvin, circa 1882 (predecessor of the Watt/Kibble balance)



Utilize new inexpensive digital scale (under $25, 0-20 gram with milligram resolution)

AWS Gemini-20

3-D printed stand with two parallel coils, radius r=2.86 cm, N=84 turns. (One rests on pedestal on scale pan, and the other is fixed in plastic stand.)



Theory: The magnetic force between two current circles, each carrying current *I*, of radius *a* and separated by a distance *c*, may be expressed in terms of the elliptic integrals of the 1st (K) and 2nd (E) kinds. Defining the modulus $k = \frac{2a}{\sqrt{4a^2+c^2}}$ and assuming superposition of *N*-turn coils, the force (SI units) is given by:

$$F(a,c,I,N) = \mu_0 N^2 I^2 \frac{c}{\sqrt{a^2+c^2}} \left[-K(k) + \frac{2a^2+c^2}{c^2} E(k)\right] \qquad (1)$$

For the spacing c << r coil radius, Eqn. (1) reduces to the familiar (algebra-based intro) expression for long parallel wires :

$$F(a,c,I,N) = \mu_0 N^2 I^2 \frac{a}{c} \qquad (2)$$

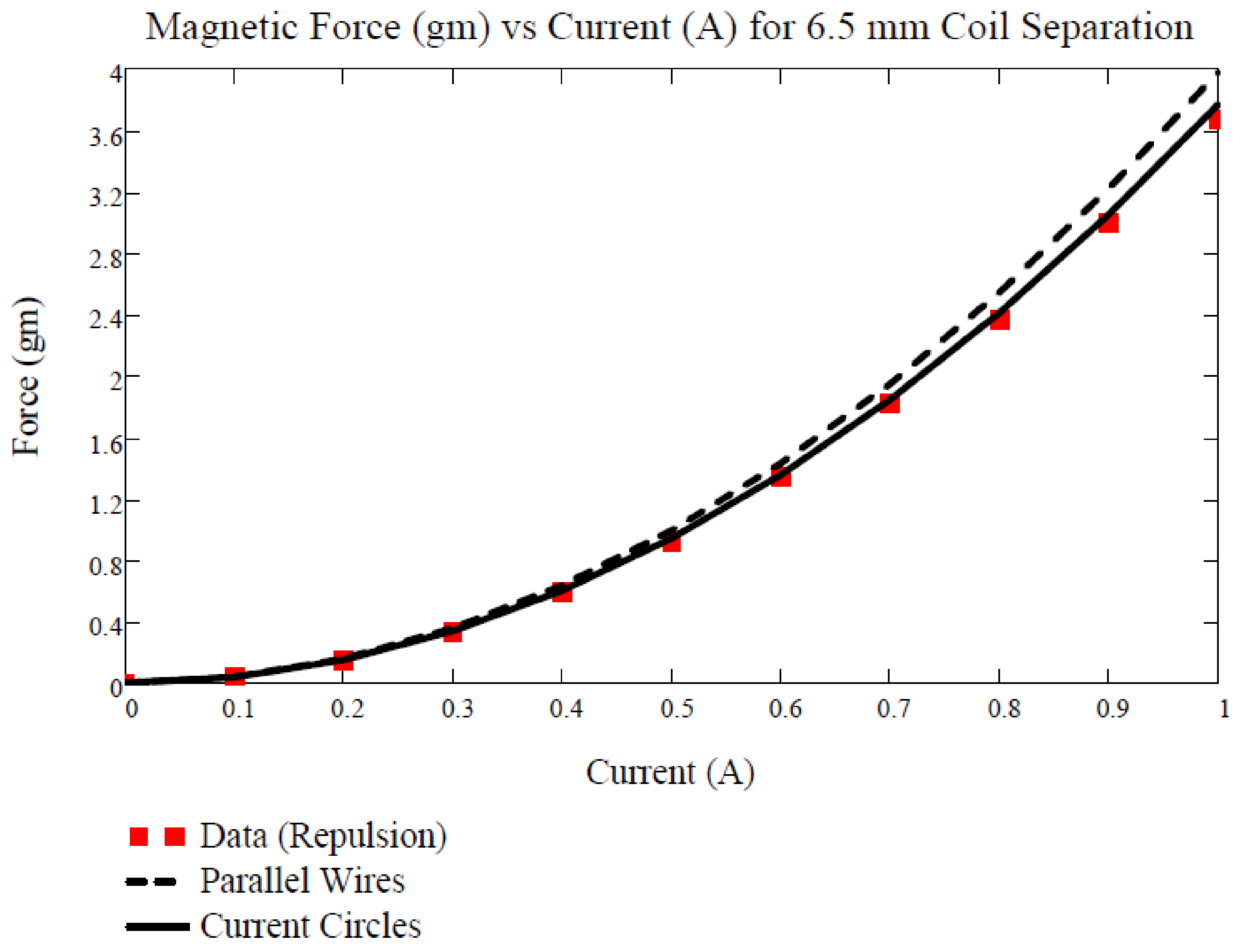
Note: Equations (2) and (3) are within 4% of each other at a separation of 5 mm for our apparatus (average radius r = 2.86 cm).

Theory (continued): Alternatively, in the limit where the spacing c>>a radius, then the fields from the coils become magnetic dipole-like and the magnetic force reduces to:

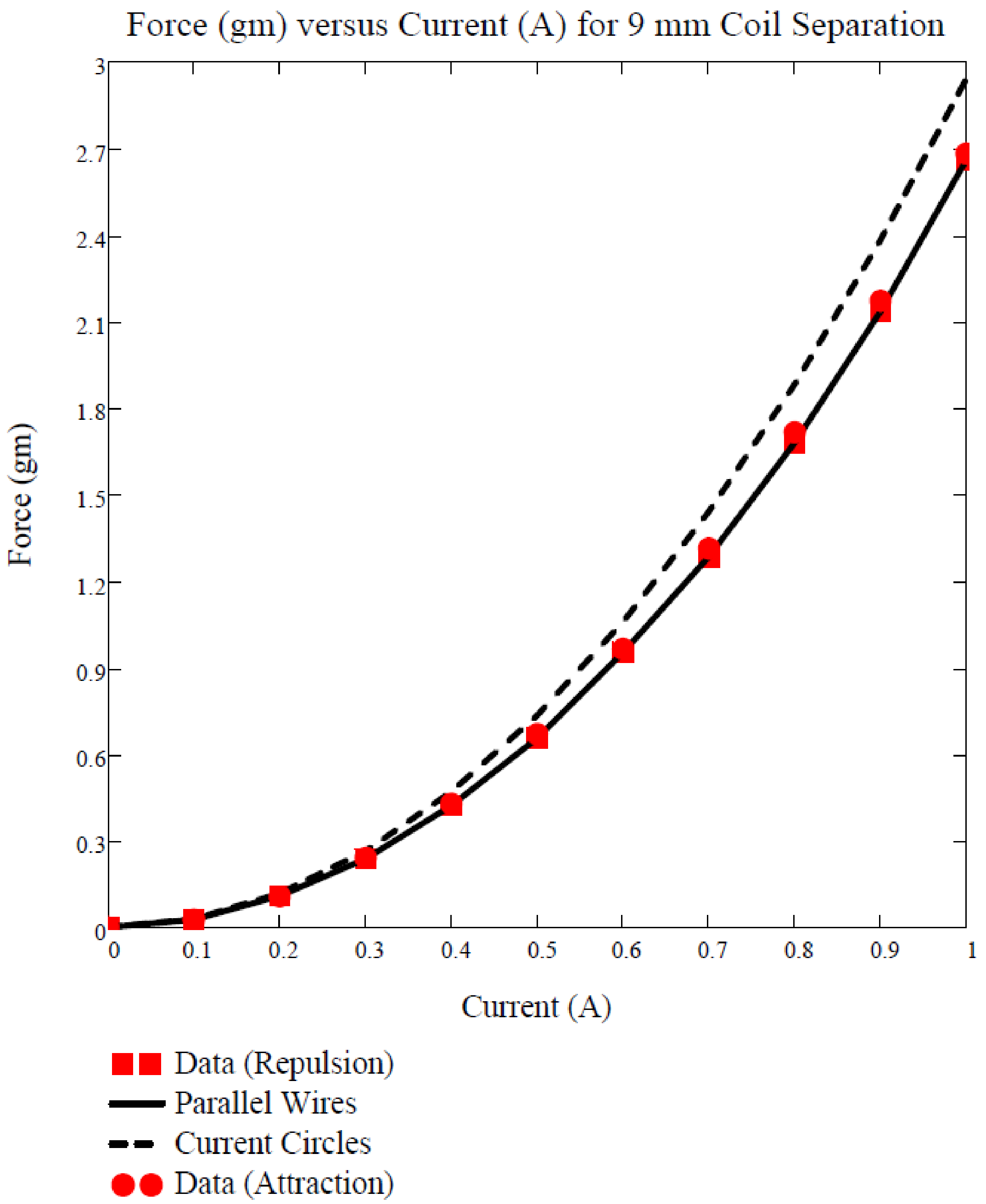
$$F(a, c, I, N) = \frac{3}{2}\pi\mu_0 N^2 I^2 \left(\frac{a}{c}\right)^4 \qquad (3)$$

(In our case, the separation c would need to be ~25 cm for the dipole force expression to become valid, so we can neglect (3)!)

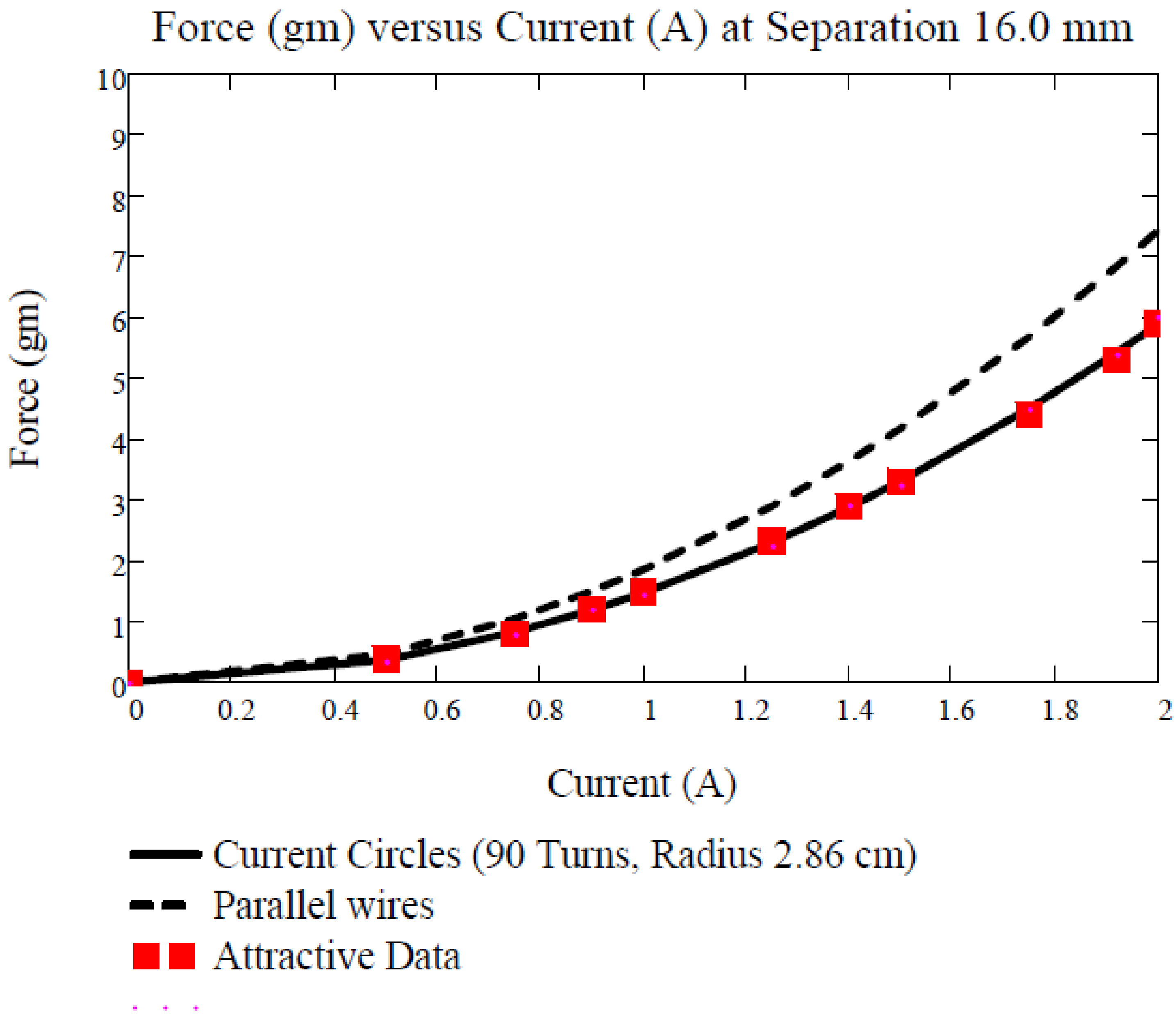
Results (Red = data, Solid line = Exact theory, Dotted line = Parallel wire approximation):



Results:


Force (gm) versus Current (A) for 9 mm Coil Separation
Force (gm)
Current (A)
0
0.3
0.6
0.9
1.2
1.5
1.8
2.1
2.4
2.7
3
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
Data (Repulsion)
Parallel Wires
Current Circles
Data (Attraction)

Results:



Force (gm) versus Current (A) at Separation 16.0 mm
Force (gm)
Current (A)
0
1
2
3
4
5
6
7
8
9
10
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2
Current Circles (90 Turns, Radius 2.86 cm)
Parallel wires
Attractive Data

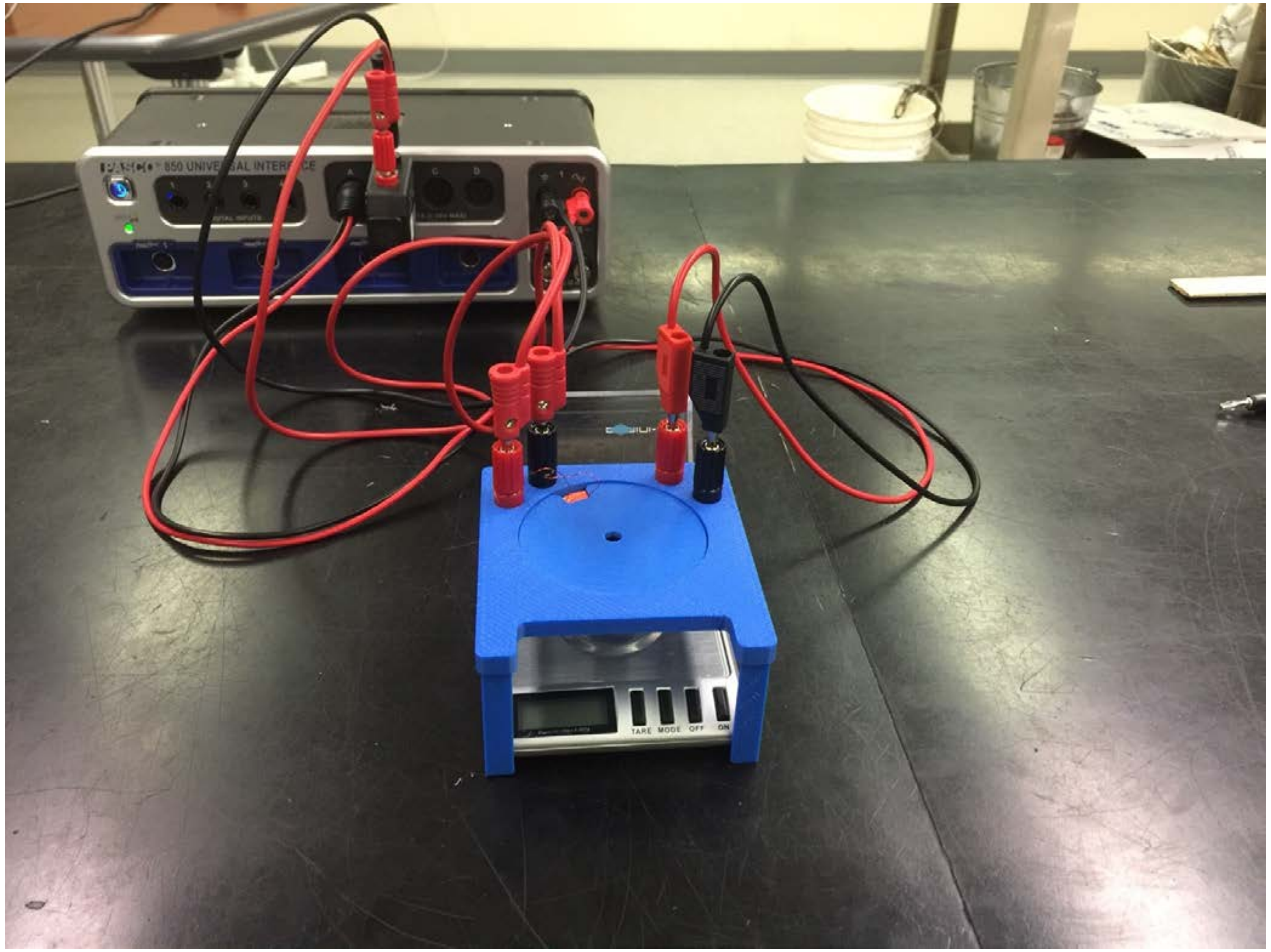
Theory (continued): The mutual inductance for parallel current circles is also given in the literature in terms of the same elliptic integrals:

$$M(a,k,N) = \mu_0 a N^2 [\left(\frac{2}{k} - k\right) K(k) - \frac{2}{k} E(k)] \quad (4)$$

It may also be measured for the parallel current circles using a signal generator and voltmeters (i.e., *Pasco 850* and *Capstone* software) and compared to theory. Excellent agreement is also obtained (6.5 mm separation)

Results: $M_{theory} = 0.41\, mH, M_{exp} = 0.43\, mH$.

Mutual Inductance Measurement:



Conclusions:

- *We have developed an accurate compact, low-cost version of the Kelvin current balance suitable for physics laboratory instruction. The coil separation may be discretely varied with a set of pedestal legs.*
- *The permeability of free space μ_0 may also be extracted.*
- *It may also be used for the quantitative measurement of mutual inductance between the coils as a function of separation.*

Thank you!

PS: The apparatus will soon be available from *Tel-Atomic.*